EUCLIDEAN GEOMETRY

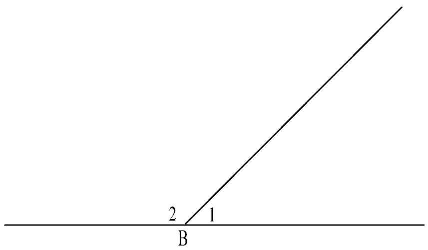
EUCLIDEAN GEOMETRY – GRADE 10

REVISION

LINES AND ANGLES

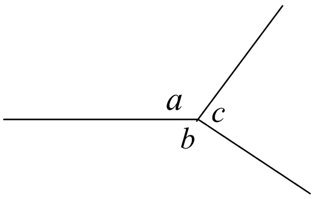
Adjacent supplementary angles

In the diagram, $\hat{B}_1 + \hat{B}_2 = 180°$

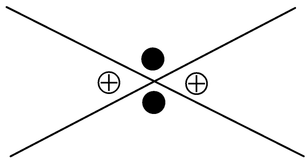


Angles round a point

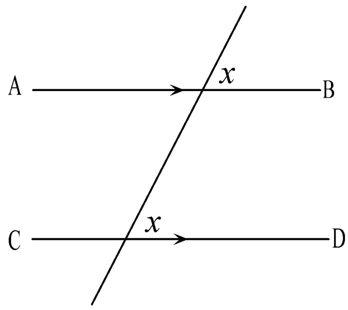
In the diagram, $a + b + c = 360°$



Vertically opposite angles
Vertically opposite angles are equal.



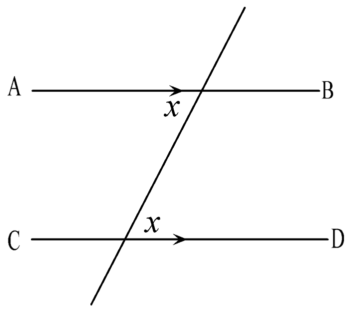
Corresponding angles
If AB||CD, then the corresponding



angles are equal.

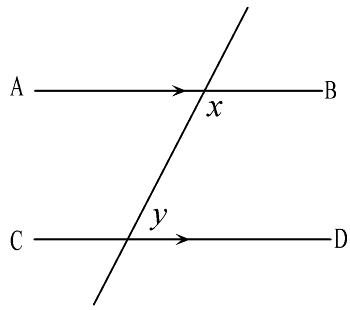
Alternate angles

If AB||CD, then the alternate angles are equal.



Co-interior angles

If AB||CD, then the co-interior angles add up to $180°$, i.e. $x + y = 180°$

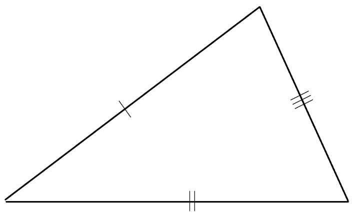


TRIANGLES

There are four kinds of triangles:

Scalene Triangle

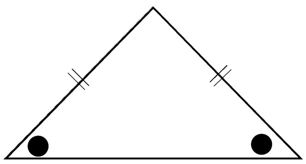
No sides are equal in length



Isosceles Triangle

Two sides are equal

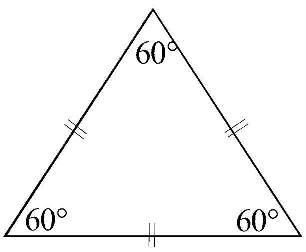
Base angles are equal



Equilateral Triangle

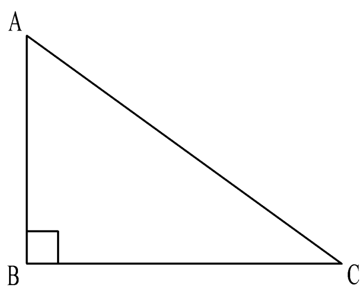
All three sides are equal

All three intericxor angles are equal

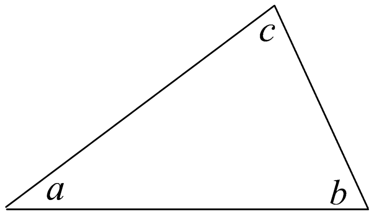


Right-angled triangle

One interior angle is $90°$

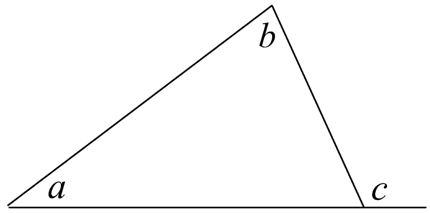


Sum of the angles of a triangle



$a + b + c = 180°$

Exterior angle of a triangle



$c = a + b$

The Theorem of Pythagoras

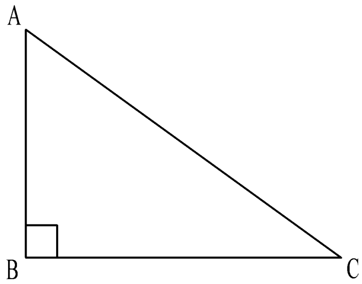
$AC^2 = AB^2 + BC^2$

or

$AB^2 = AC^2 - BC^2$

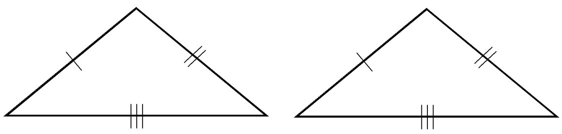
or

$BC^2 = AC^2 - AB^2$

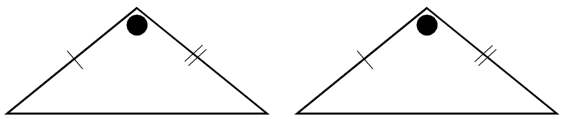


Congruency of triangles (four conditions)

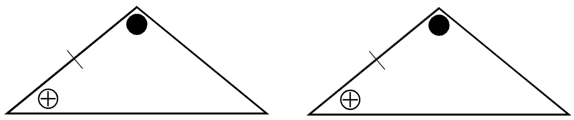
Condition 1
Two triangles are congruent if three sides of one triangle are equal in length to the three sides of the other triangle.



Condition 2
Two triangles are congruent if two sides and the included angle are equal to two sides and the included angle of the other triangle.

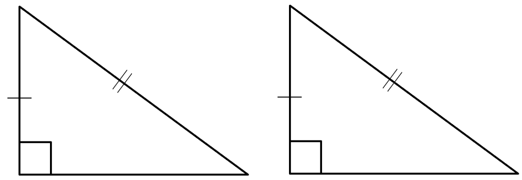


Condition 3
Two triangles are congruent if two angles and one side are equal to two angles and one side of the other triangle.



Condition 4

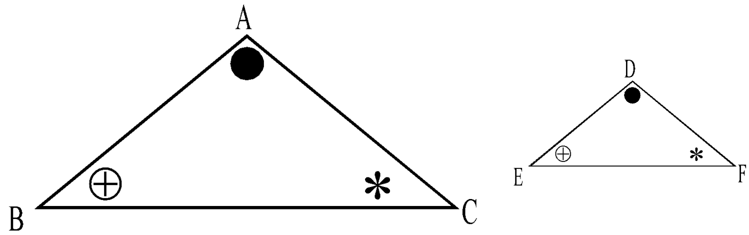
Two right-angled triangles are congruent if the hypotenuse and a side of the one triangle is equal to the hypotenuse and a side of the other triangle.



Similar Triangles

If two triangles are similar (equiangular), then their corresponding sides are in the same proportion.

If $\Delta ABC ||| \Delta DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



SOLVING GEOMETRIC RIDERS

STEP 1
- **Analyse** the diagram given.

STEP 2
- Analyse the diagram by paying attention to **key words** given.
- Look for information in the diagram which can be **helpful and useful**
- **Use colours** to mark off equal angles / sides
- Look for **implied information**.

STEP 3
- Brainstorm and develop a **rough proof.**
- **Link** the information you have acquired

STEP 4
- Rewrite a **formal proof**

EXAMPLES OF KEY WORDS

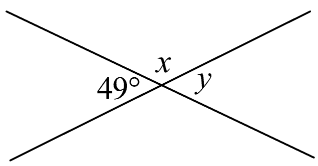
- Parallel lines given

- Triangle information
- How to prove that lines are parallel?
- Centre of circle given
- Diameter given
- Angles on the same segment
- Chords
- Cyclic quads
- Tangents etc.

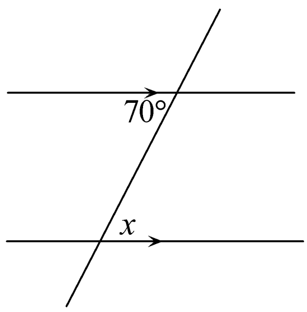
<u>EXERCISE 1</u> **(Revision of Grade 8 and 9)**

1. Calculate the size of the angles marked with small letters:

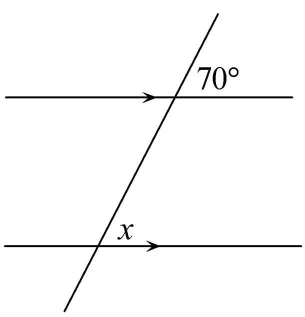
(a)



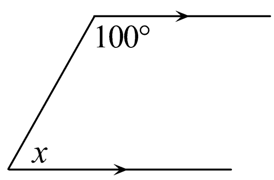
(b)



(c)

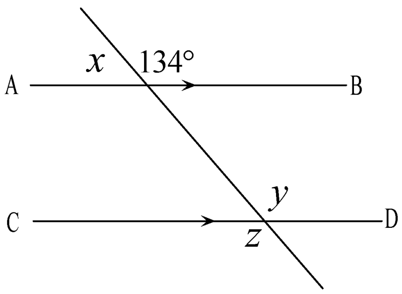


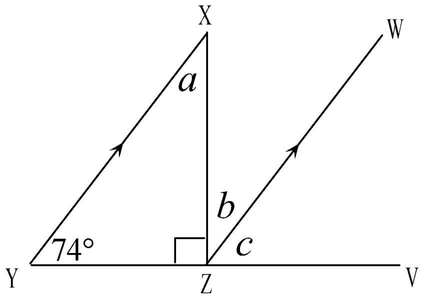
(d)



(e)

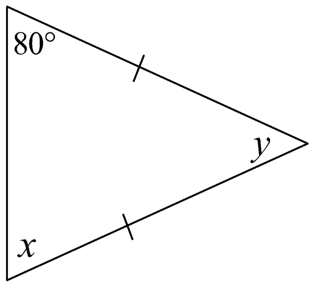
(f)



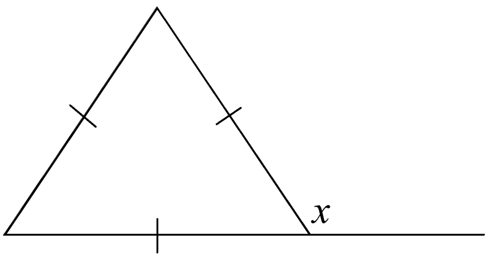


(

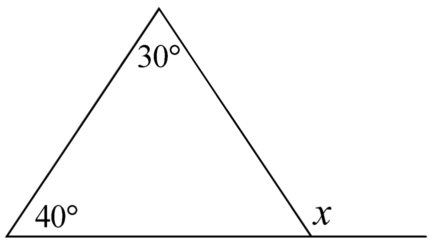
(g)



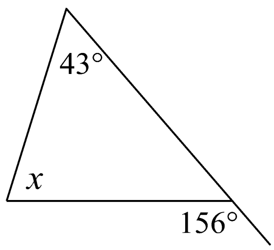
(h)



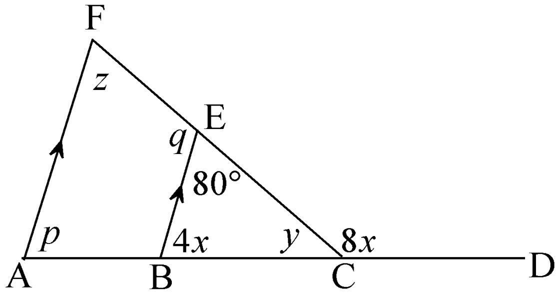
(i)



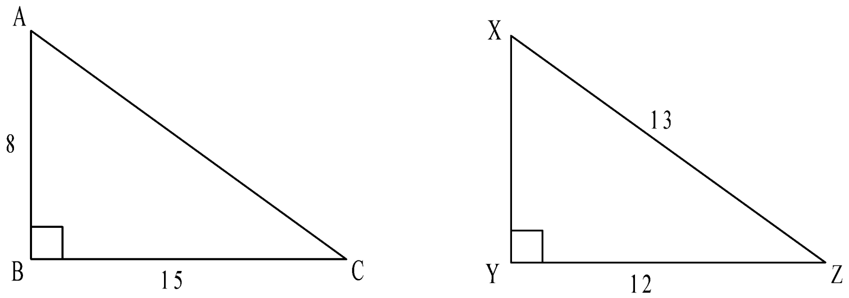
(j)



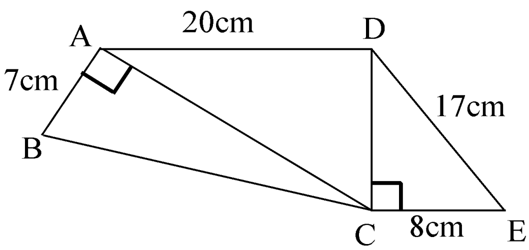
2. In ΔACF, AF||BE. Calculate, with reasons, the sizes of all angles indicated by a small letter.



3. (a) Calculate AC. (b) Calculate XY.

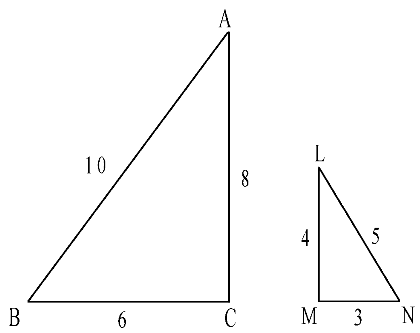


(c) Calculate the length of BC.

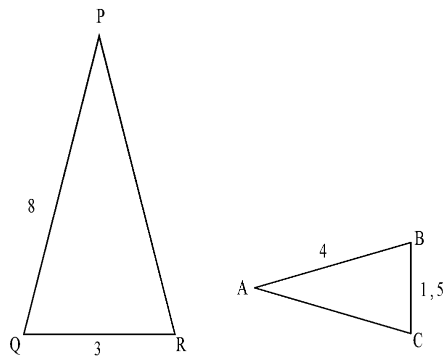


4. Are the following pairs of triangles similar? Give a reason for your answer.

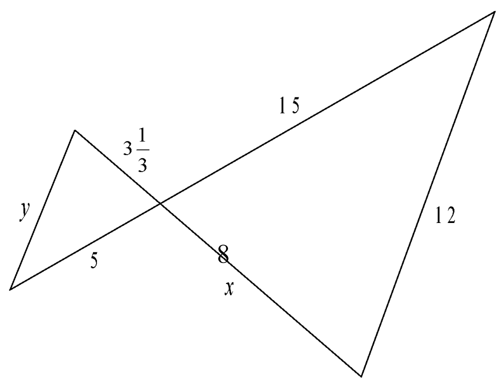
(a)



(b)



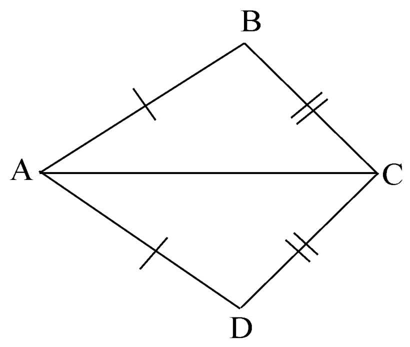
5. The two triangles below are similar. Calculate the value of *x* and *y*.



6. ABCD is a kite in which $AB = AD$ and $BC = CD$.
Prove that:

(a) $\Delta ABC \equiv \Delta ADC$

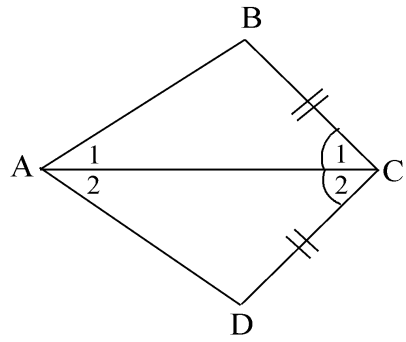
(b) Why is $\hat{B} = \hat{C}$?



7. ABCD is a kite in which $\hat{C}_1 = \hat{C}_2$ and $BC = CD$.
Prove that:

(a) $\Delta ABC \equiv \Delta ADC$

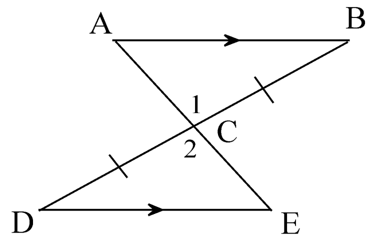
(b) Why is $\hat{A}_1 = \hat{A}_2$?



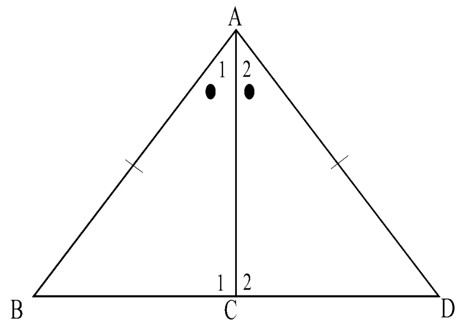
8. AB||DE and $DC = CB$. Prove that:

(a) $AC = CE$

(b) $AB = DE$

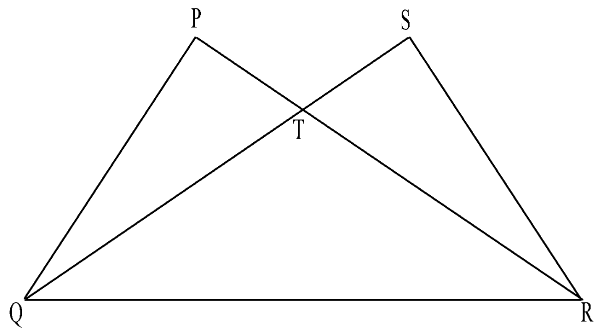


9. Prove that $\Delta ABC \equiv \Delta ADC$ using two different conditions of congruency.

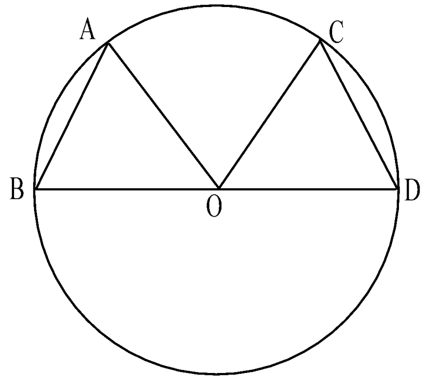


10. In the figure below, sides PR and QS of triangles PQR and SQR intersect at T. $PQ = SR$ and $\hat{P} = \hat{S} = 90°$.

Prove that $\Delta PQR \equiv \Delta SQR$.



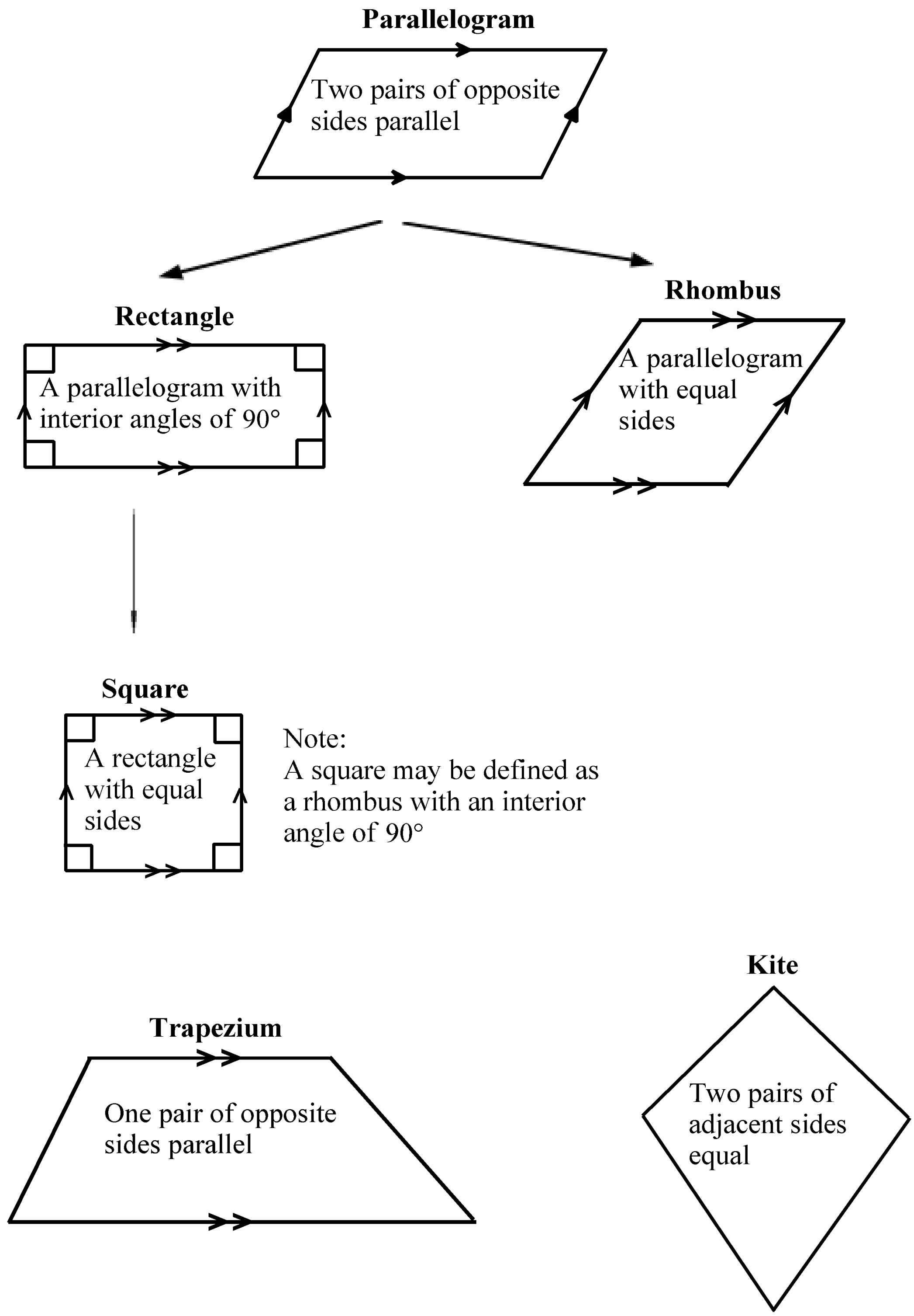
11. Prove that $\Delta AOB \equiv \Delta COD$ if O is the centre of the circle and $AB = CD$.



QUADRILATERALS

A **polygon** is a two-dimensional figure with three or more straight sides. A **quadrilateral** is a polygon with four straight sides.

Definitions of quadrilaterals

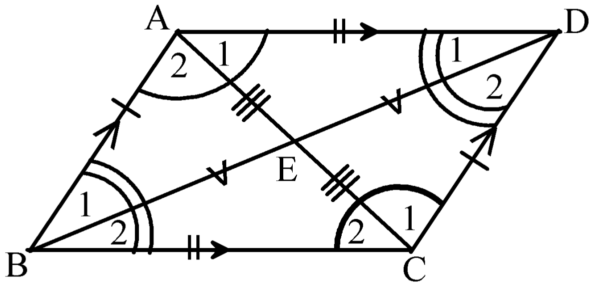


PROPERTIES OF QUADRILATERALS

(It is advisable to first do the investigation and mind map on quadrilaterals before discussing the properties which follow – see Teacher's Guide)

Parallelogram

If ABCD is a parallelogram, you may assume the following properties:



AD||BC ; AB||DC

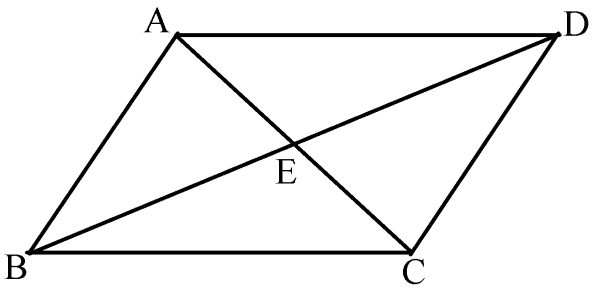
AD = BC ; AB = DC

AE = EC ; BE = ED

$\hat{D}_1 = \hat{B}_2$; $\hat{D}_2 = \hat{B}_1$; $\hat{C}_1 = \hat{A}_2$; $\hat{C}_2 = \hat{A}_1$

$\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

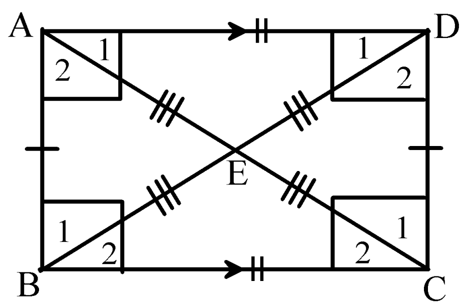
In order to prove that a quadrilateral is a parallelogram, you will need to prove at least one of the following:



AD||BC and AB||DC	Opp sides ||
AD = BC and AB = DC	Opp sides =
AE = EC and BE = ED	Diagonals bisect
$\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$	Opp angles =
AB||DC and AB = DC	
AD||BC and AD = BC	

Rectangle

If ABCD is a rectangle, you may assume the following properties:



AD||BC ; AB||DC

AD = BC ; AB = DC

AE = EC = BE = ED

$\hat{D}_1 = \hat{B}_2$; $\hat{D}_2 = \hat{B}_1$; $\hat{C}_1 = \hat{A}_2$; $\hat{C}_2 = \hat{A}_1$

$\hat{A} = \hat{C} = \hat{B} = \hat{D} = 90°$

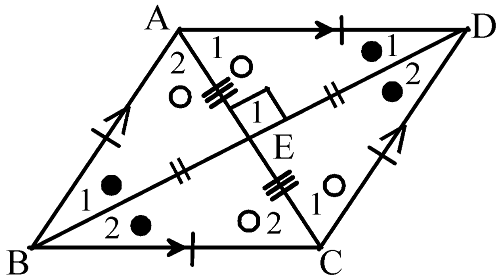
In order to prove that a quadrilateral is a rectangle, you will need to prove one of the following:

(a) The quadrilateral is a parallelogram with at least one interior angle equal to $90°$.

(b) The diagonals of the quadrilateral are equal in length and bisect each other.

Rhombus

If ABCD is a rhombus, you may assume the following properties:



AD||BC ; AB||DC

AD = BC = AB = DC

AE = EC ; BE = ED

$\hat{D}_1 = \hat{D}_2 = \hat{B}_1 = \hat{B}_2$

$\hat{A}_1 = \hat{A}_2 = \hat{C}_1 = \hat{C}_2$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

$\hat{E}_1 = 90°$; $AC \perp BD$

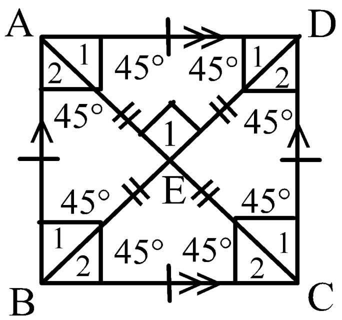
In order to prove that a quadrilateral is a rhombus, you will need to prove one of the following:

(a) The quadrilateral is a parallelogram with a pair of adjacent sides equal

(b) The quadrilateral is a parallelogram in which the diagonals bisect at right angles.

Square

If ABCD is a square, you may assume the following properties:



AD||BC ; AB||DC

AD = BC = AB = DC

AE = EC = BE = ED

$\hat{D}_1 = \hat{D}_2 = \hat{B}_1 = \hat{B}_2 = \hat{A}_1 = \hat{A}_2 = \hat{C}_1 = \hat{C}_2 = 45°$

$\hat{A} = \hat{C} = \hat{B} = \hat{D} = 90°$

$\hat{E}_1 = 90°$; $AC \perp BD$

In order to prove that a quadrilateral is a square, you will need to prove one of the following:

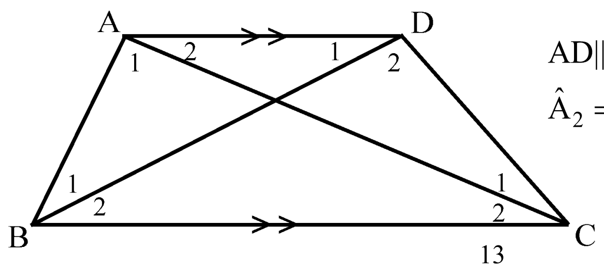
(a) The quadrilateral is a parallelogram with an interior right angle and a pair of adjacent sides equal.

(b) The quadrilateral is a rhombus with an interior right angle

(c) The quadrilateral is a rhombus with equal diagonals.

Trapezium

If ABCD is a trapezium, you may assume the following properties:



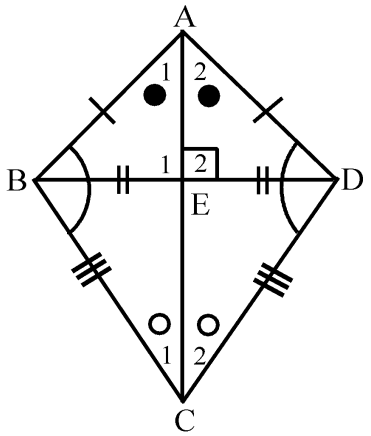
AD||BC

$\hat{A}_2 = \hat{C}_2$; $\hat{D}_1 = \hat{B}_2$

In order to prove that a quadrilateral is a trapezium, you will need to prove that AD||BC.

Kite

If ABCD is a kite, you may assume the following properties:



AB = AD

BC = DC

BE = ED

$\hat{A}_1 = \hat{A}_2$

$\hat{C}_1 = \hat{C}_2$

$\hat{B} = \hat{D}$

$\hat{E}_2 = 90°$

$AC \perp BD$

In order to prove that a quadrilateral is a kite, you will need to prove that the pairs of adjacent sides are equal in length.

GRADE10 ATP

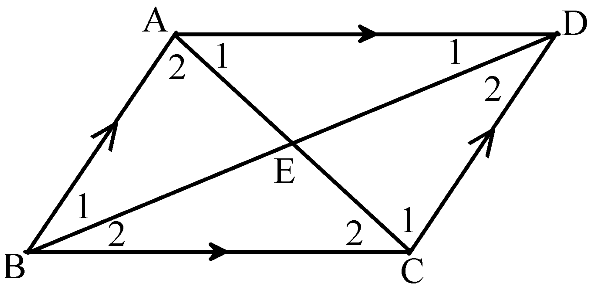
THEOREM 1

The opposite sides and angles of a parallelogram are equal.

Required to prove: $AB = CD$; $AD = BC$; $\hat{A} = \hat{C}$; $\hat{B} = \hat{D}$

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.



In ΔABC and ΔCDA :

(a) $\hat{A}_2 = \hat{C}_1$ alt angles equal

(b) $\hat{A}_1 = \hat{C}_2$ alt angles equal

(c) $AC = AC$ common side

$\therefore \Delta ABC \equiv \Delta CDA$ SAA

$\therefore AB = CD$ and $AD = BC$

Also $\hat{B} = \hat{D}$

Similarly, it can be proved that $\Delta ABD \equiv \Delta CDB$

$\therefore \hat{A} = \hat{D}$

CONVERSE OF THEOREM 1

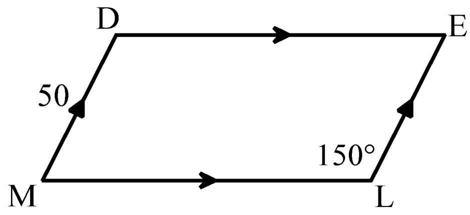
(a) If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.

(b) If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.

EXAMPLE 1

DELM is a parallelogram with $DM = 50$ and $\hat{L} = 150^\circ$. Calculate the length of EL and the sizes of $\hat{D}, \hat{E}$ and $\hat{M}$.

Solution

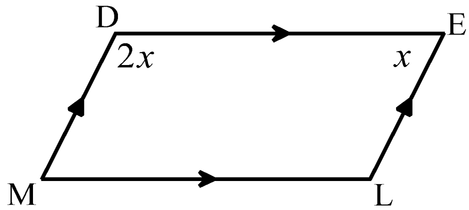


$EL = 50$	Opp sides of parm
$\hat{D} = 150°$	Opp angles of parm
$\hat{E} = 30°$	Co-interior angles
$\hat{M} = 30°$	Opp angles of parm

EXAMPLE 2

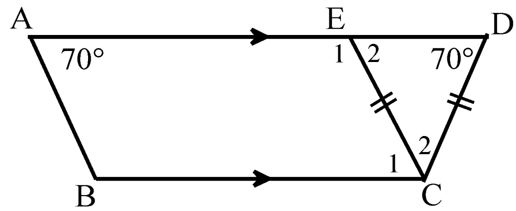
DELM is a parallelogram. Calculate the value of x and hence the sizes of the interior angles.

Solution



$2x + x = 180°$	Co-interior angles
$\therefore 3x = 180°$	
$\therefore x = 60°$	
$\therefore \hat{E} = 60°$	
$\therefore \hat{M} = 60°$	Opp angles of parm
$\hat{D} = 2(60°) = 120°$	
$\hat{L} = 120°$	Opp angles of parm

EXAMPLE 3



In trapezium ABCD, AD||BC with $\hat{A} = \hat{D} = 70°$ and $EC = DC$. Prove that ABCE is a parallelogram.

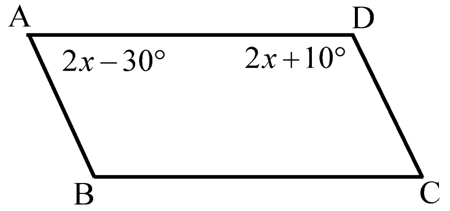
Solution

$\hat{E}_2 = 70°$	Angles opp equal sides
$\hat{C}_1 = 70°$	Alt angles equal
$\therefore \hat{A} = \hat{C}_1$	
$\hat{E}_1 = 110°$	Adjacent suppl angles
$\hat{B} = 110°$	Co-interior angles
$\therefore \hat{E}_1 = \hat{B}$	

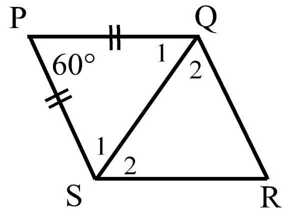
Therefore, ABCE is a parallelogram Both pair of opposite angles equal

EXERCISE 2

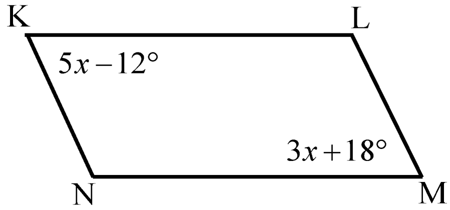
1. Determine the sizes of the interior angles of parallelogram ABCD.



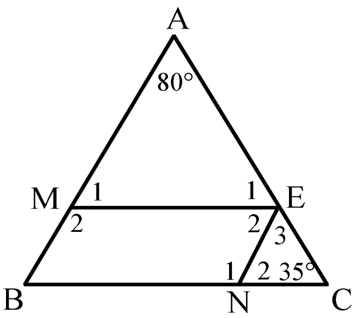
2. PQRS is a parallelogramwith $\hat{P}=60°$ and $PQ=PS$. Calculate the sizes of $\hat{R}$, $\hat{S}_1$, $\hat{Q}_1$, $\hat{Q}_2$ and $\hat{S}_2$.



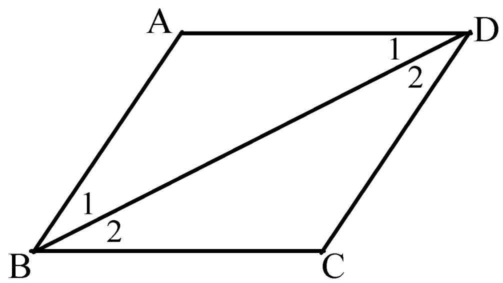
3. KLMN is a parallelogram. Calculate the size of the interior angles.



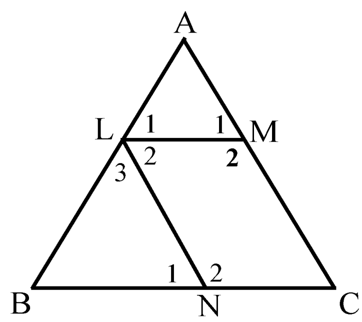
4. In ΔABC, $\hat{A}=80°$ and $\hat{C}=35°$. Calculate the interior angles of parallelogram MENB.



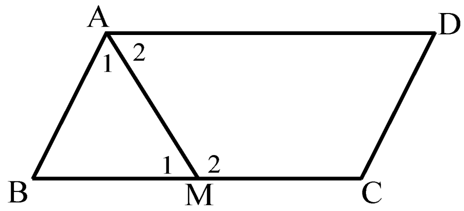
5. In parallelogram ABCD, $AB=AD$ and $\hat{C}=110°$. Calculate the size of all interior angles.



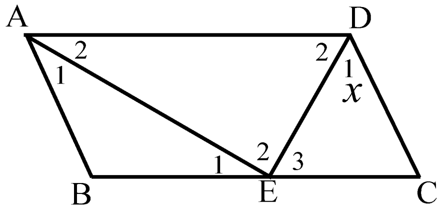
6. ΔABC is an equilateral triangle. Determine the interior angles of parallelogram LMCN.



7. ABCD is a parallelogram. AM bisects $\hat{A}$. $AB = AM$. $\hat{C} = 120°$. Calculate the sizes of all interior angles.



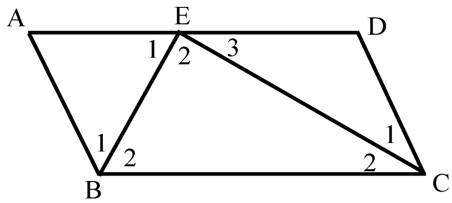
8. In parallelogram ABCD, $AB = BE = DE$ $\hat{A}_1 = 28°$ Calculate the size $\hat{D}_1$ if $\hat{D}_1 = x$.



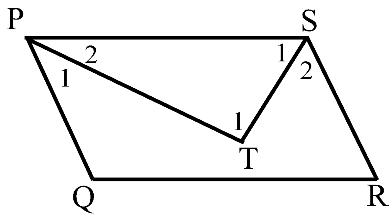
9. In parallelogram ABCD, $\hat{A} = 120°$, $AB = 50cm$ and E is a point on AD such that $AB = AE$ and $CD = DE$. Determine:

(a) DE

(b) the perimeter of ABCD.



10. In parallelogram PQRS, $\hat{Q} = 114°$, PT bisects $\hat{P}$ and TS bisects $\hat{S}$. Prove that $PT \perp ST$.

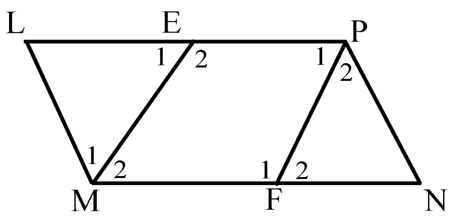


11. In quadrilateral LMNP, $\hat{E}_1 = 62°$, $\hat{P}_1 = 68°$, $\hat{P}_2 = 56°$, $FP = FN$ and $LE = LM$.

Prove that:

(a) LP||MN

(b) LMNP is a parallelogram.

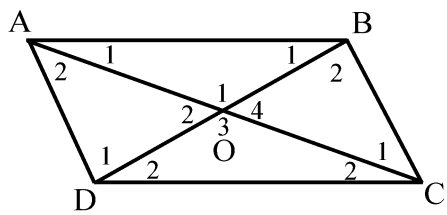


12. AC and DB are diagonals of quadrilateral ABCD. $AO = OC$ and $BO = OD$. Prove:

(a) $\Delta AOD \equiv \Delta COB$

(b) $\Delta AOB \equiv \Delta COD$

(c) ABCD is a parallelogram.



THEOREM 2

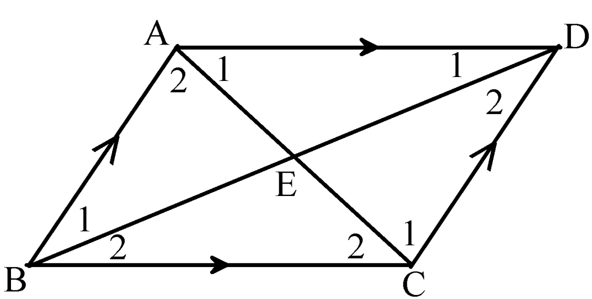
The diagonals of a parallelogram bisect each other.

Required to prove: AE = EC and BE = ED

Proof

Draw parallelogram ABCD and join the diagonals AC and BD.

In ΔABE and ΔCDE :



(a) $\hat{A}_2 = \hat{C}_1$ alt angles equal

(b) $\hat{B}_1 = \hat{D}_2$ alt angles equal

(c) $AB = DC$ opp sides of parm

$\therefore \Delta ABE \equiv \Delta CDE$ SAA

$\therefore BE = ED$ and $AE = EC$

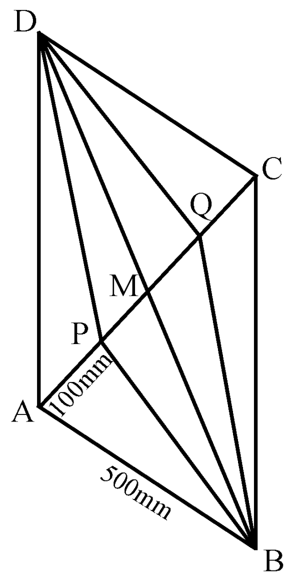
CONVERSE OF THEOREM 2

If the diagonals of quadrilateral bisect each other, then the quadrilateral is a parallelogram.

EXAMPLE 4

Diagonals AC and BD of parallelogram ABCD intersect at M. $AP = QC$ and $AC = 600mm$, $AB = 500mm$ and $AP = 100mm$. Prove that PBQD is a parallelogram.

Solution



$AM = MC$ diagonals of a parm

But $AC = 600mm$ given

$\therefore AM = MC = 300mm$

$AP = QC = 100mm$ given

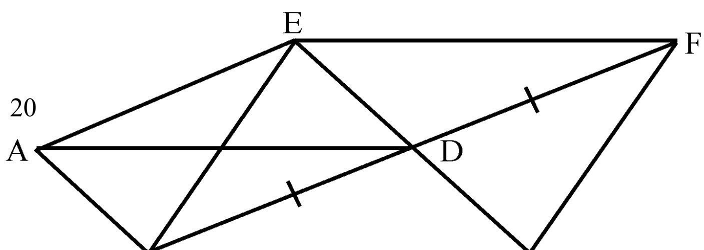
$\therefore PM = MQ = 200mm$

Also $BM = MD$ diagonals of parm

$\therefore PM = MQ$ and $BM = MD$

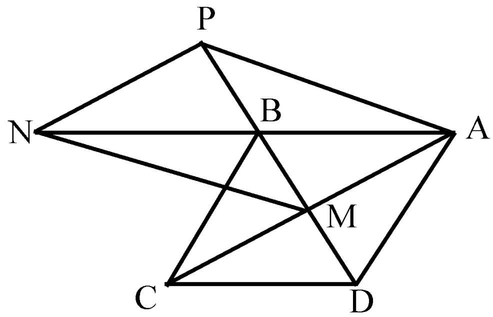
$\therefore$ PBQD is a parallelogram since diagonals bisect each other.

EXERCISE 3

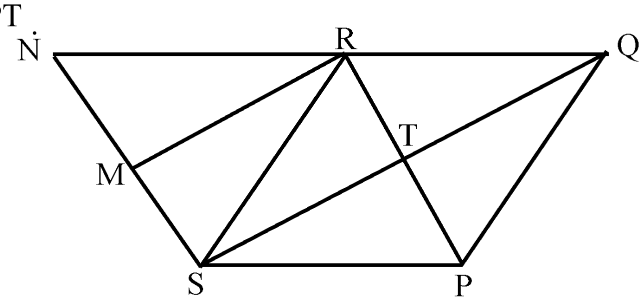


1. Parallelograms ABCD and ABDE are given with $DF = DB$.
Prove that BCFE is a parallelogram.

2. ABCD is a parallelogram. $DM = BP$ and $DC = BN$. Prove:
(a) APNM is a parallelogram.
(b) $PN = MC$



3. PQRS is a parallelogram. PR and QS intersect at T. $QT = RM$ and $SM = PT$.
Prove that:
(a) RTSM is a parallelogram.
(b) $QR = RN$



THEOREM 3

If two opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.

Required to prove: ABCD is a parallelogram

Proof:

In ΔABC and ΔCDA :

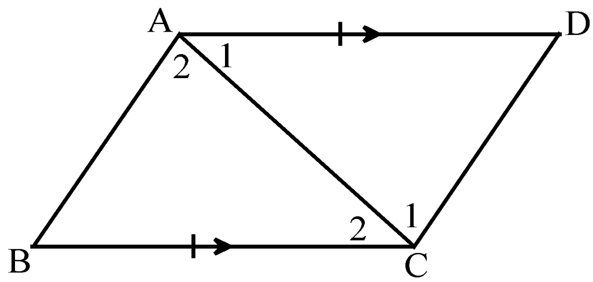
(a) $\hat{A}_1 = \hat{C}_2$ alt angles equal

(b) $AC = AC$ commom side

(c) $AD = BC$ given

$\therefore \Delta ABC \equiv \Delta CDA$ SAS

$\therefore AB = DC$



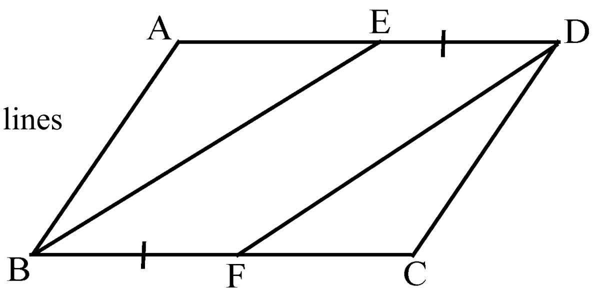
and $AD = BC$

$\therefore$ ABCD is a parallelogram since the opposite sides are equal.

EXAMPLE 5

ABCD is a parallelogram with $ED = BF$. Prove that BEFD is a parallelogram.

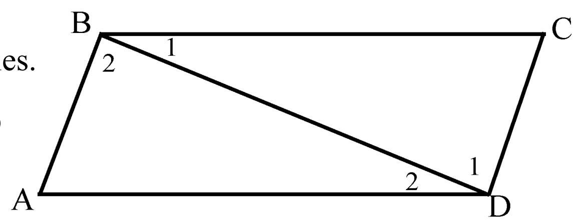
Solution



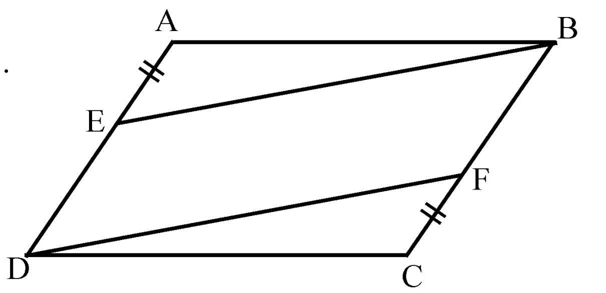
$AD \parallel BC$	opp sides parm parallel
$\therefore ED \parallel BF$	AED and BFC are straight lines
and $ED = BF$	given
$\therefore$ BEFD is a parm	one pair of opp sides equal and parallel.

EXERCISE 4

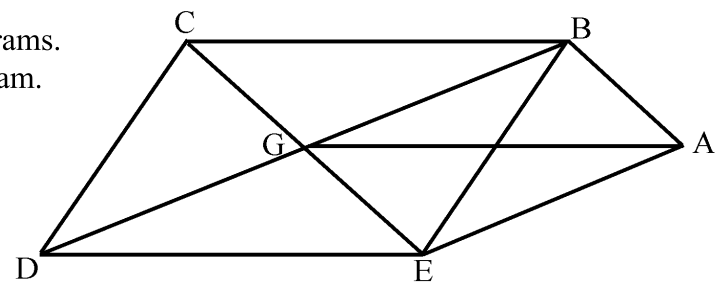
1. ΔABD and ΔBCD are two isosceles triangles. $\hat{C} = 75°$ and $A\hat{D}B = 30°$. Prove that ABCD is a parallelogram.



2. ABCD is a parallelogram with $AE = FC$. Prove that BEDF is a parallelogram.



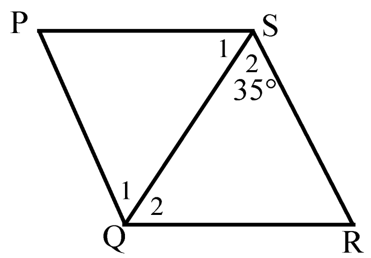
3. BCDE and ABCG are parallelograms. Prove that ABGE is a parallelogram.



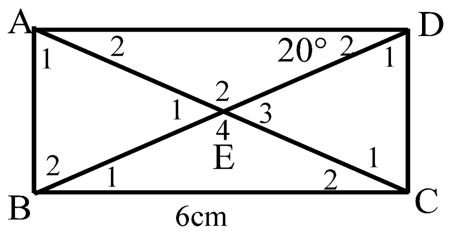
The next exercise involves the properties of rectangles, rhombuses and squares. Familiarise yourself with the properties of these quadrilaterals before attempting the exercises.

<u>EXERCISE 5 (Rectangles, rhombuses, squares, trapeziums and kites)</u>

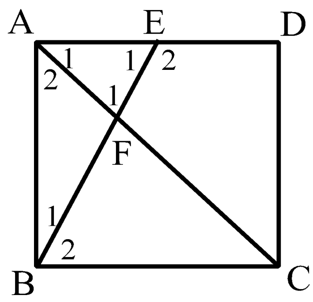
1. PQRS is a rhombus with $\hat{S}_2 = 35°$.
   Calculate the size of all other interior angles.



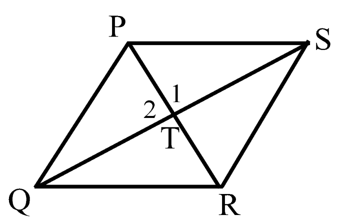
2. Diagonals AC and BD intersect at E.
   ABCD is a rectangle with $AC = 10cm$ and $BC = 6cm$. $\hat{D}_2 = 20°$.
   Calculate the following:
   $\hat{A}_1$; $\hat{A}_2$; $\hat{B}_1$; $\hat{B}_2$; $\hat{C}_1$; $\hat{C}_2$; $\hat{D}_1$,
   AD, AE and AB.



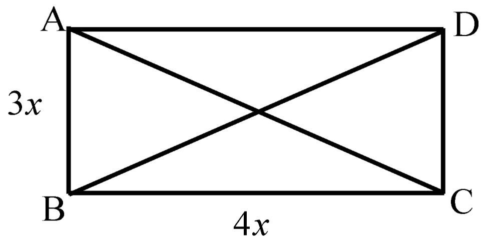
3. ABCD is a square. $A\hat{E}B = 55°$.
   Calculate $\hat{F}_1$.



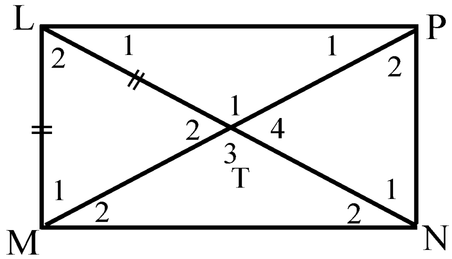
4. In rhombus PQRS, $PQ = 26cm$ and $QS = 48cm$.
   Calculate the length of PR.



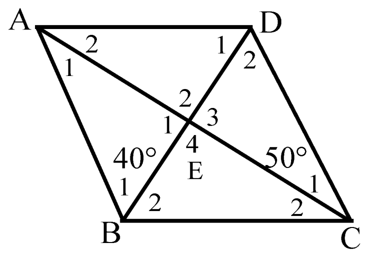
5. In rectangle ABCD, $AB = 3x$ and $BC = 4x$. Find the length of AC and BD in terms of x.



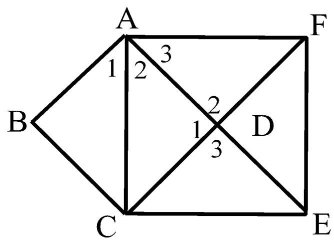
6. The diagonals of parallelogram LMNP intersect at T. $LT = LM$ and $M\hat{T}N = 120°$. Prove that LMNP is a rectangle.



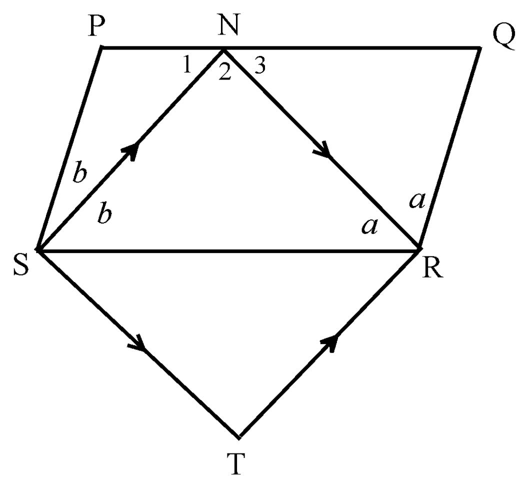
7. ABCD is a parallelogram. $\hat{B}_1 = 40°$ and $\hat{C}_1 = 50°$. Prove that ABCD is a rhombus.



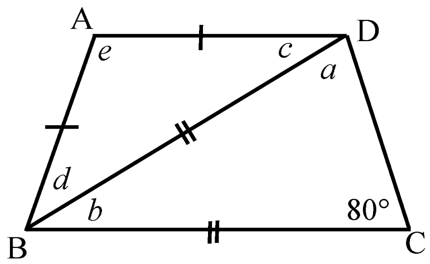
8. ABCD is a square. $DE = DA$ and $DF = DC$. Prove that ACEF is a square.



9. In parallelogram PQRS, NR bisects $S\hat{R}Q$ and NS bisects $P\hat{S}R$. SN||RT and NR||ST Prove that STRN is a rectangle.



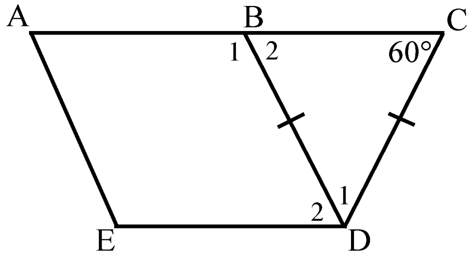
10. ABCD is a trapezium with AD||BC. $AB = AD$ and $BD = BC$. $\hat{C} = 80°$. Determine the unknown angles.



11. ABCDE is an isosceles trapezium. $BC = CD$ and $\hat{C} = 60°$. Prove that:

(a) ABDE is a parallelogram.

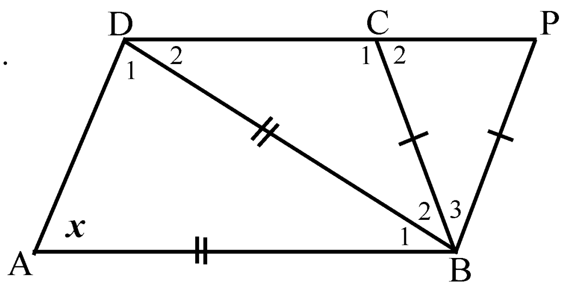
(b) ΔBCD is equilateral.



12. ABCD is an isosceles trapezium with $\hat{A} = x$. $BC = BP$ and $AB = DB$. Prove that:

(a) $\hat{P} = x$

(b) $\Delta ABD \equiv \Delta PDB$

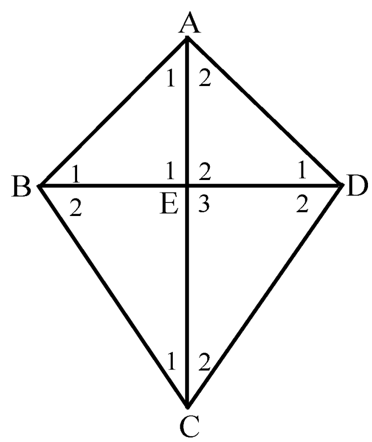


13. ABCD is a kite. The diagonals intersect at E. $BD = 30\text{cm}$, $AD = 17\text{cm}$ and $DC = 25\text{cm}$. Determine:

(a) AE

(b) AC

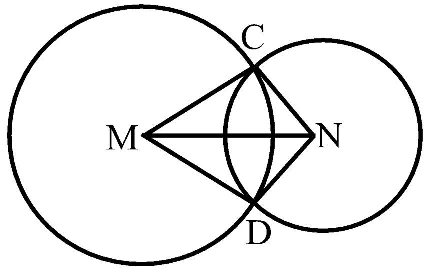
(c) $\hat{B}_1$ if $\hat{A}_1 = 20°$



14 Circle centre M intersects circle centre N at C and D. Prove that:

(a) MDNC is a kite.

(b) $M\hat{C}N = M\hat{D}N$

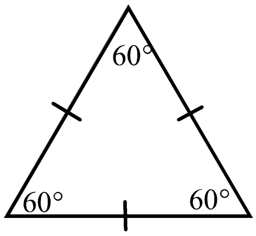
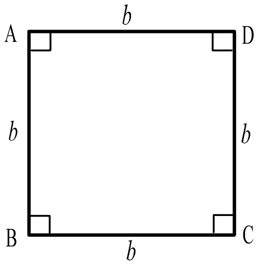
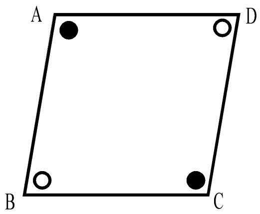
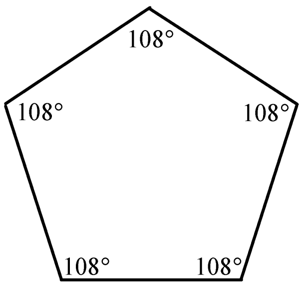


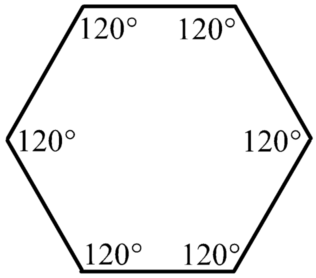
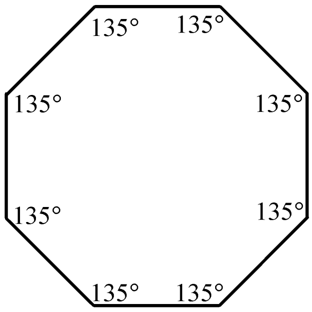
MORE ON POLYGONS

A polygon is a two-dimensional figure with three or more straight sides. A **regular polygon** is a polygon in which all the sides are equal in length.

- The rule for calculating the sum of the interior angles of a polygon of n sides is given by the formula: $180°(n-2)$
- The size of an interior angle of a regular polygon is given by the formula: $\frac{180°(n-2)}{n}$

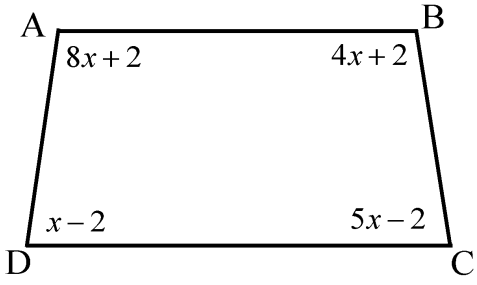
Polygons	**Name**	**Interior angles**

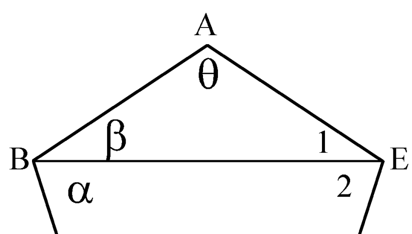
3  **sides**	**Equilateral Triangle (Regular)**	The sum of the interior angles: $180°(3-2)=180°$ The size of an interior angle: $\frac{180°(3-2)}{3}=60°$
4  **sides**	**Square (Regular)**	The sum of the interior angles: $180°(4-2)=360°$ The size of an interior angle: $\frac{180°(4-2)}{4}=90°$
4 sides 	**Rhombus (Not regular)**	The sum of the interior angles: $180°(4-2)=360°$. However, the opposite angles are equal but the co-interior angles add up to $180°$ (not all angles are equal as in other regular polygons).
 **5 sides**	**Pentagon (Regular)**	The sum of the interior angles: $180°(5-2)=540°$. The size of an interior angle: $\frac{180°(5-2)}{5}=108°$

6 sides 	**Hexagon (Regular)**	The sum of the interior angles: $180°(6-2)=720°$. The size of an interior angle: $\frac{180°(6-2)}{6}=120°$
8 sides 	**Octagon (Regular)**	The sum of the interior angles: $180°(8-2)=1080°$. The size of an interior angle: $\frac{180°(8-2)}{8}=135°$

EXERCISE 6

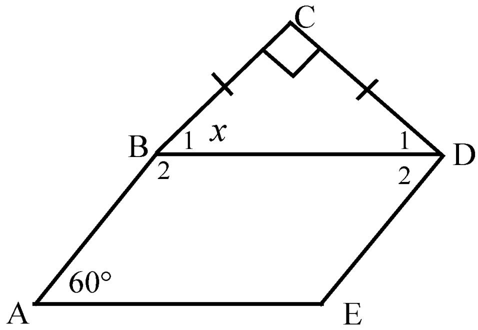
1. ABCD is a polygon with four sides.
   (a) Calculate the value of x.
   (b) Hence show that ABCD is a trapezium.



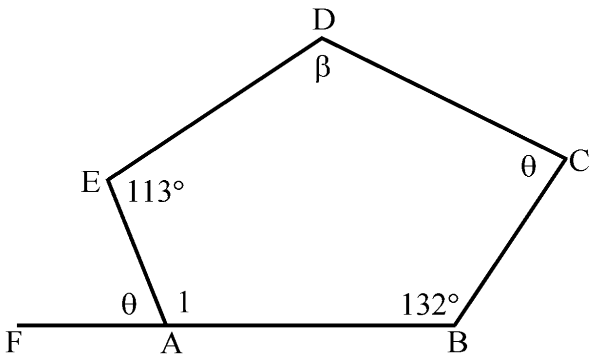


2. ABCD is a pentagon made up of five equal sides and five equal interior angles. Calculate the size of θ, β and α .

3. In polygon ABCDE, $\hat{C} = 90°$, $BC = CD$ and ABDE is a parallelogram. Use **TWO** different methods to determine the value of x.

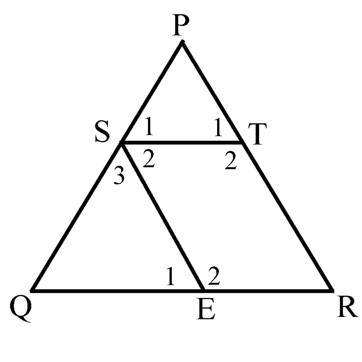


4. Using the information provided on the diagram, determine β .



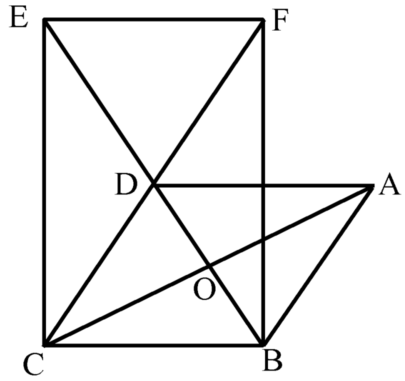
<u>**MIXED REVISION EXERCISE**</u>

1. In ΔPQR , $PQ = PR$ and STRE is a

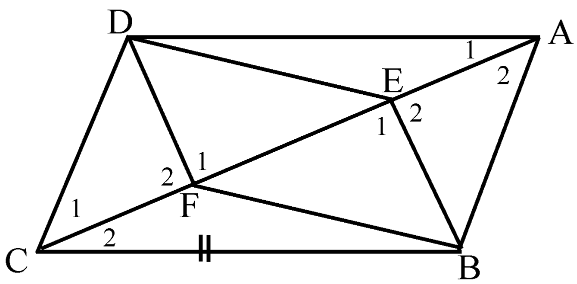


Parallelogram. $\hat{Q} = x$ and $\hat{P} = 2\hat{Q}$.
Calculate the sizes of the angles of STRE.

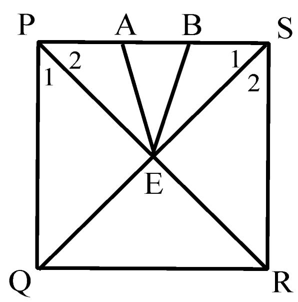
2. ABCD is a parallelogram. $FD = DC$ and $DE = 2DO$.
$DO = x$. Prove that BCEF is a Parallelogram.



3. ABCD is a parallelogram. $BE \perp AC$ and $DF \perp AC$. Prove that EBFD is a parallelogram.

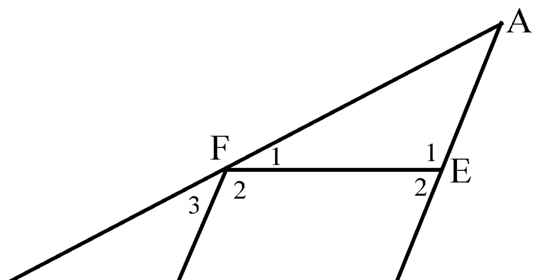


4. PQRS is a square. The diagonals intersect at E. $PA = BS$. Prove that ΔAEB is an isosceles triangle.



SOME CHALLENGES

1. FCDE is a parallelogram. CE is produced



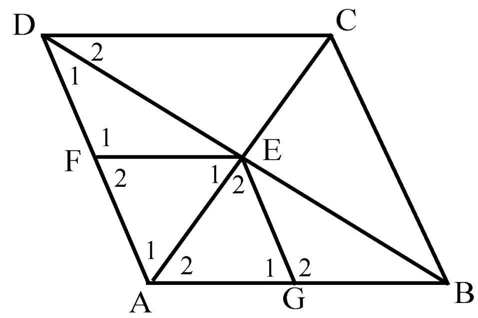
to A such that $CE = EA$ and $CD = DB$.

Prove that:

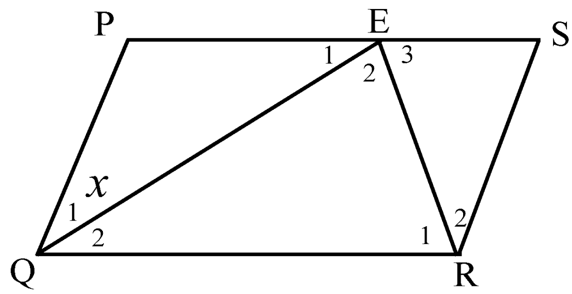
(a) $\Delta BDF \equiv \Delta FEA$

(b) BFA is a straight line (Hint: prove that $\hat{F}_1 + \hat{F}_2 + \hat{F}_3 = 180°$)

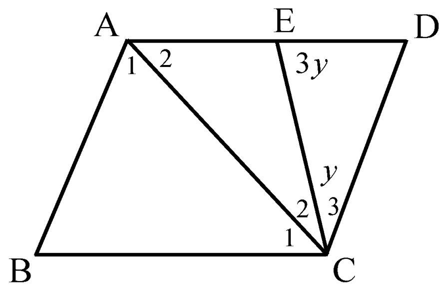
2. ABCD is a rhombus. Diagonals intersect at E. $EF = FA$ and $EG = GA$. Prove that AGEF is a rhombus.



3. PQRS is a parallelogram. $PQ = PE$, $QE = QR$, $ER = SR$ and $P\hat{Q}E = x$. Calculate the size of $Q\hat{E}R$.



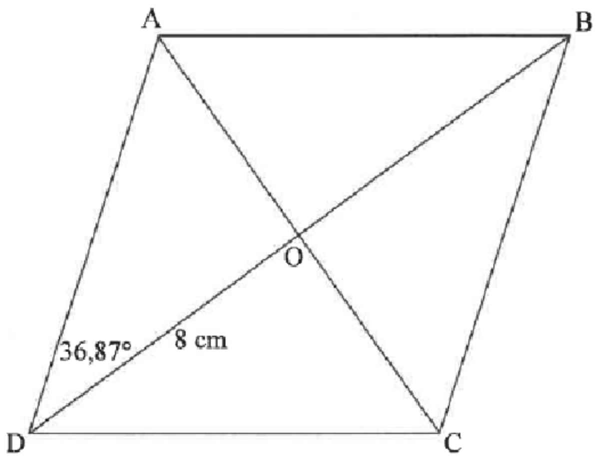
4. ABCD is a rhombus. $D\hat{E}C = 3y$ and $\hat{C} = y$. Prove that EC bisects $A\hat{C}D$.



DBE,2015 November Paper 2 Grade 10

QUESTION 8

In the diagram, ABCD is a rhombus having diagonals AC and BD intersecting in O. $A\hat{D}O = 36{,}87°$ and DO = 8 cm.



8.1 Write down the sizes of the following angles:

8.1.1 $C\hat{D}O$

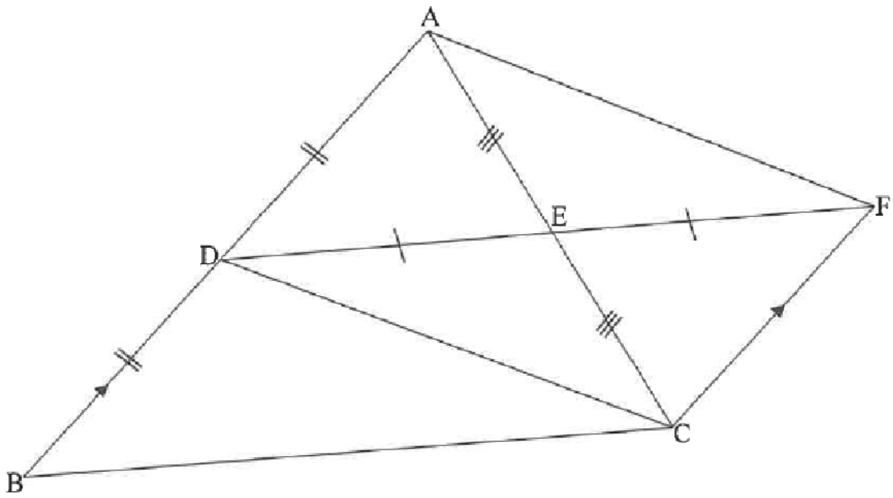
8.1.2 $A\hat{O}D$

8.2 Calculate the length of AO.

8.3 If E is a point on AB such that OE | | DA, calculate the length of OE.

QUESTION 9

9.1 In the diagram below, D is the midpoint of side AB of ΔABC. E is the midpoi of AC. DE is produced to F such that DE = EF. CF | | BA.

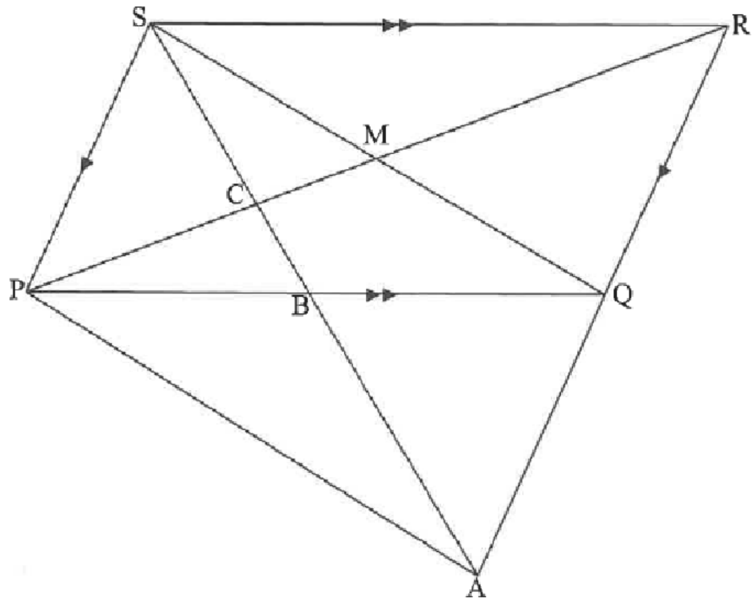


9.1.1 Write down a reason why $\Delta ADE \equiv \Delta CFE$.

9.1.2 Write down a reason why DBCF is a parallelogram.

9.1.3 Hence, prove the theorem which states that $DE = \frac{1}{2}BC$.

9.2 In the diagram below, PQRS is a parallelogram having diagonals PR and intersecting in M. B is a point on PQ such that SBA and RQA are straight and SB = BA. SA cuts PR in C and PA is drawn.



9.2.1 Prove that SP = QA.

9.2.2 Prove that SPAQ is a parallelogram.

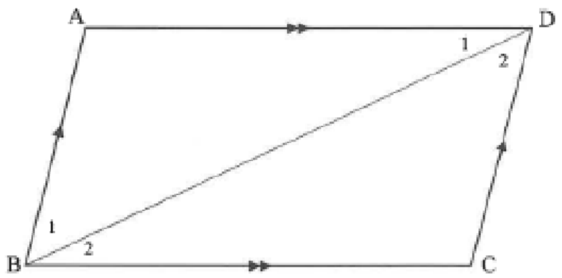
9.2.3 Prove that AR = 4MB.

QUESTION 8

8.1 Complete the following statement:

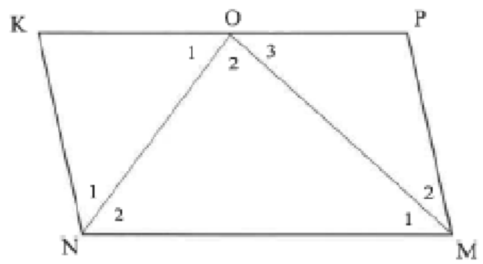
If the opposite angles of a quadrilateral are equal, then the quadrilateral … (1)

8.2 Use the sketch below to prove that the opposite sides of a parallelogram are equal.



(6)

8.3 In the sketch below, KPMN is a parallelogram. ON bisects $K\hat{N}M$ and OM bisects $N\hat{M}P$.



8.3.1 Show that $N\hat{O}M = 90°$. (3)

8.3.2 Prove that O is the midpoint of KP. (6)

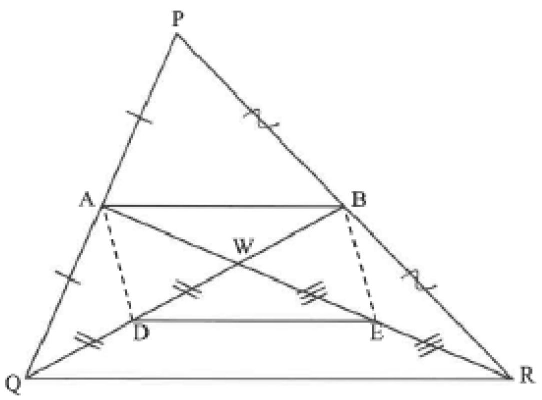
[16]

QUESTION 9

9.1 Complete the following statement:

The line through the midpoint of two sides in a triangle is parallel to and … the third side. (1)

9.2 In ΔPQR, A and B are the midpoints of sides PQ and PR respectively. AR and BQ intersect at W. D and E are points on WQ and WR respectively such that $WD = DQ$ and $WE = ER$.



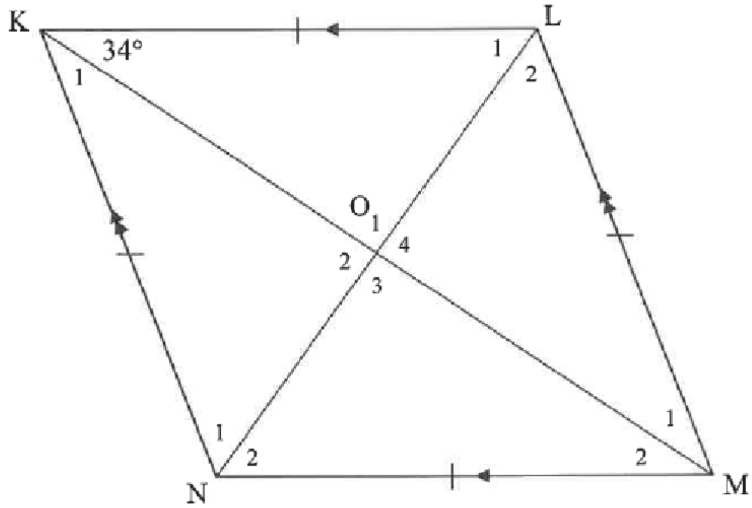
Prove that ADEB is a parallelogram. (5)

[6]

DBE,2017 November Paper 2 Grade 10

QUESTION 8

8.1 KLMN is a rhombus with diagonals intersecting at O. $L\hat{K}M = 34°$.

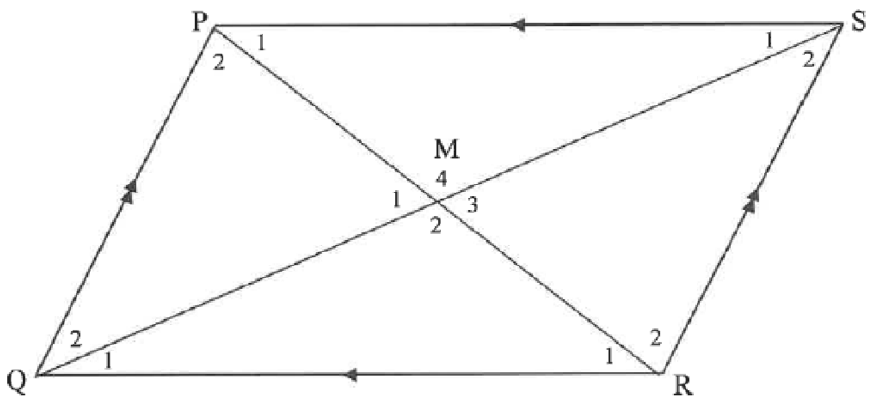


8.1.1 Write down the size of $\hat{O}_1$.

8.1.2 Calculate the size of $\hat{L}_1$.

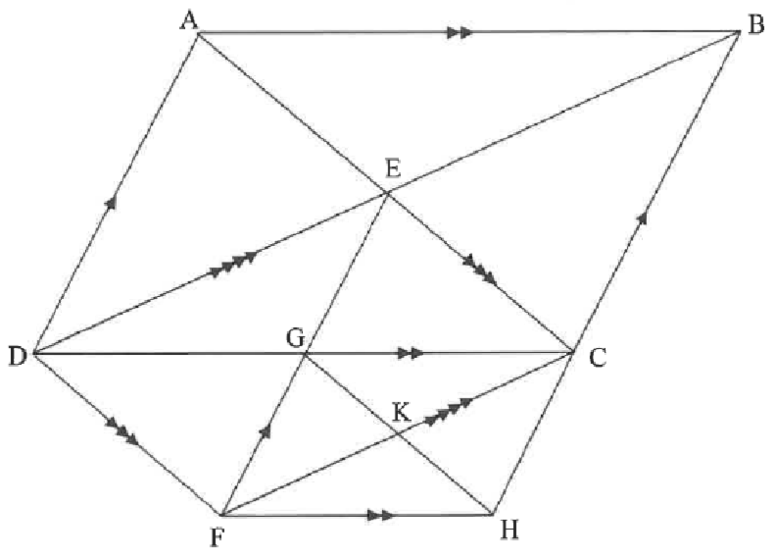
8.1.3 Calculate the size of $K\hat{N}M$.

8.2 Given parallelogram PQRS with diagonals PR and QS intersecting at M.



Prove that the diagonals bisect each other.

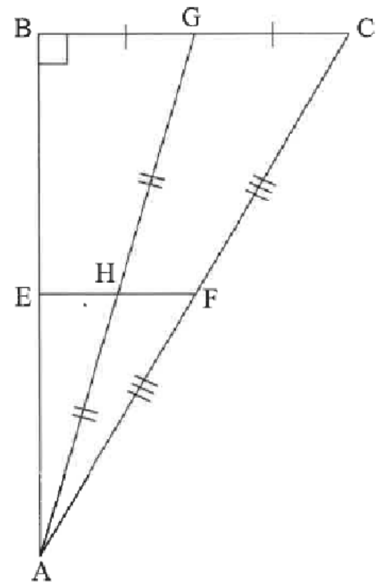
8.3 In the diagram, ABCD is a parallelogram with diagonals intersecting at E. The diagonals of parallelogram DECF intersect at G. The diagonals of parallelogram FGCH intersect at K.



Prove that DB = 4KC.

QUESTION 9

ΔABC is right-angled at B. F and G are the midpoints of AC and BC respectively. H is the midpoint of AG. E lies on AB such that FHE is a straight line.



9.1 Prove that E is the midpoint of AB.

9.2 If $EH = 3,5$ cm and the area of $\Delta AEH = 9,5 cm^2$, calculate the length of AB.

9.3 Hence, calculate the area of ΔABC.